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A new turbulence model for porous media flows. Part I: Constitutive equations and model closure

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ABSTRACT

A new model for turbulent flows in porous media is developed. The spatial- and time fluctuations in this new model are tied together and treated as a single quantity. This novel treatment of the fluctuations leads to a natural construction of the k and ε type equations for rigid and isotropic porous media in which all the kinetic energy filtered in the averaging process is modeled. The same terms as those found in the corresponding equations for clear flow, plus additional terms resulting from the interaction between solid walls in the porous media and the fluid characterize the model. These extra terms arise in a boundary integral form, facilitating their modeling. The model is closed by assuming the eddy viscosity approximation to be valid, and using simple models to represent the interaction between the walls in the porous media and the fluid.

1. Introduction

The analysis of fluid flowing through porous media is required in a large range of applications in such industries as chemical, mechanical, nuclear, geological, environmental, petroleum, etc. The pore-size's spectrum is vast and can vary from the order of Å (ultra-micro-pores) to cms (pebbles, food) or even larger [1]. Moreover, the conditions encountered in different applications are broad enough to cover a large range of Reynolds numbers. For example, Stokes type flows in porous media may be encountered in ground water flows while turbulent flows are found in application such as heat exchangers or nuclear reactors. Due to lack of geometric information to model each and every pore, such systems are difficult to simulate with full geometric details. Despite the fact that it might be possible to describe some of these systems in an almost exact representation or a meaningful statistical approximation of the geometry, the computational effort required to solve the flow field in such geometries is still out of reach. This motivates the research in the development of porous media approximation, representing the system composed of pores by a macroscopic homogenous one with uniform properties. While both, laminar and turbulent, flows are important [2–7], number of modeling efforts reported for turbulent flows in porous media is relatively small.

An example of a system where porous media approximation seems to be a suitable approach is the Pebble Bed Nuclear Reactor. This system has been usually modeled as a homogeneous system using porous media approximation [2,8]. The operating conditions in this reactor are characterized by high Reynolds numbers or turbulent flows ($Re_p \approx 13{,}000$, based on average pore velocity and average pore diameter, [8]) together with large changes in the temperature of the cooling gas ($\Delta T \approx 500\ ^\circ C$). These operating conditions make necessary the use of models that account for both compressibility and turbulence effects.

Different flow regimes have been identified in porous media flows. Dybbs and Edwards [9] classified four different flow regimens depending on the pore Reynolds number. For example, unsteady and chaotic flow is encountered for $Re_p > 300$. In porous media, turbulence based on *microscopic*, or point-wise quantities, is not fundamentally different from turbulence in clear flows. Most approaches to model turbulent flow in porous media are based on the approaches usually followed to model turbulence in clear flows [10–13]. Not surprisingly, the most common approach is to extend the k–ε model. Since a region of space occupied by solid and fluid, in the porous media approximation is represented by a "homogenized" region, results obtained using such models are relevant only at the macroscopic scale. Space-fluctuating quantities in addition to the Reynolds decomposition are then introduced in the macroscopic representation [14].

Turbulence (k–ε type) models for porous media developed earlier differ from each other because they are based on different definitions of the macroscopic turbulence quantities, such as the turbulent kinetic energy and the dissipation rate [15]. The intro-

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Nomenclature

A_i	fluid–solid interface inside the REV
D	macroscopic length scale
f	model function (macroscopic turbulent dissipation rate equation)
F	Forchheimer coefficient
k	macroscopic turbulent kinetic energy (MTKE)
K	permeability
l_m	macroscopic mixing length scale
L	megascale
p	pore length scale
R_i	Darcy–Forchheimer drag force
Re	Reynolds number
Re_p	Reynolds number based on the average pore velocity and average pore dimension
$Re_{\sqrt{K}}$	Reynolds number based on the fluid averaged velocity and permeability
U	intrinsic fluid velocity
V	volume of the REV
V_f	fluid volume inside the REV
V_s	solid volume inside the REV

Greek symbols

ε	macroscopic turbulent dissipation rate (MDR)
ε_m	turbulent dissipation rate
ϕ	porosity
ν	kinematic fluid viscosity
ν_T	macroscopic eddy viscosity
ρ	fluid density

Additional notations

$\langle\psi\rangle$	V-normalized space average of ψ
$\langle\psi\rangle^f$	V_f -normalized space average of ψ
${}^i\psi$	space fluctuation of ψ
$\overline{\psi}$	used for time average of ψ or for V_f -normalized space–time average of ψ
$\underline{\psi}'$	time fluctuation of ψ
$\overline{\underline{\psi}}$	V_f -normalized space–time average of ψ
$\overline{\psi}^v$	V-normalized space–time average of ψ
ψ''	space–time fluctuation of ψ

duction of different macroscopic or space-averaged quantities lead to different spatial correlations (in addition to the well known time fluctuations found in turbulent flows in clear media) that need to be modeled. Unfortunately, relevant microscopic experimental data is scarce, and hence any comparative analysis of these models and space-averaged quantities must be based on an evaluation of the assumptions employed in the development, simplicity of the model, and, when available on comparison of macroscopic results predicted by the models with experimental data.

There is no consensus in the research community as to which approach to model turbulence effects in porous media is most suitable (see for instance [16,17]). Lack of agreement on this topic has opened up space for additional research and new modeling approaches to develop better understanding of the phenomenon of turbulence in porous media, or, in other words, to develop better representation of turbulence within the framework of the porous media approximation. Therefore, the aim of this work is to develop an alternative turbulence model for flow in porous media.

The approach followed here in developing a new model for turbulent flow in porous media [18,19] is based on redefining the turbulence quantities, and consequently their transport equations in the k–ε turbulence model, in such a way that time and space fluctuations are not specifically distinguished. These new definitions of the turbulence quantities lead to a model in which all of the kinetic energy filtered in the space–time-averaging process is modeled in its transport equation. Moreover, the resulting model is *simple*: the definition leads to a natural construction of the k and ε equations, with the same terms found in the corresponding equations for clear flow, plus additional (boundary) terms resulting from the interaction between solid walls of the porous media and the fluid.

Organization of the paper is as follows. An introduction to the space or volume-averaging approach generally used in modeling porous media is given. A brief review of existing turbulence models for flow in porous media is then presented and aspects of these models that may need further improvement are identified. Seeking to overcome these aspects, a new set of equations is developed by treating the average over space- and time fluctuations as a unique quantity. The new set of equations is closed, thus developing a one-equation model and a two-equation model to describe turbulent effects at macroscopic scales in porous media. The second part of this study – validating the model, contrasting it with existing results and numerical simulations – is presented in an accompanying paper (Part II) [20].

2. Macroscopic variables and space-averaged quantities

The notion of space average in porous media is based on the assumption that although fluid velocities and pressure may be irregular at the pore scale, locally space-averaged measurements of these quantities vary smoothly [21]. Macroscopic equations are commonly obtained by spatially averaging the microscopic ones over a *representative elementary volume* (REV) of the porous medium. A schematic representation of a spherical REV consisting of a fixed solid phase saturated with a continuous fluid phase is shown in Fig. 1 (note that the solid phase is fixed, that is, the solid phase does not change randomly if different ensembles are considered). The volume of the REV is constant (no space dependence) and its value is equal to the sum of the fluid and solid volumes inside the REV ($V = V_f + V_s$). For averaging purposes, an auxiliary coordinate system $\underline{r} = \underline{x} + \underline{\xi}$ is defined, so that $\underline{x}$ describes the origin of each averaging volume and $\underline{\xi}$ is the position in a local coordinate system specific to each averaging volume. In this treatment, the ideas of Hassanizadeh and Gray [22] and Gray et al. [23] are followed.

Defining a distribution function $\gamma_f(\underline{r})$, as one in the fluid phase and zero in the solid phase,

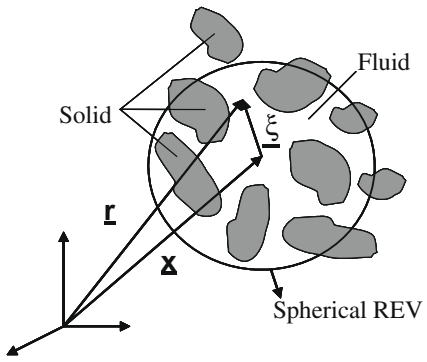

Fig. 1. Spherical representative elementary volume (REV).

$$\gamma_f(\underline{r}) = \begin{cases} 1 & \text{if } \underline{r} \in V_f \\ 0 & \text{if } \underline{r} \in V_s \end{cases}, \tag{1}$$

the cell-average $<\psi>$, and the intrinsic phase cell-average $<\psi>^f$ of any quantity $\psi(\underline{r},t)$ associated with the fluid can be defined respectively as:

$$\langle\psi\rangle(\underline{x},t) = \frac{1}{V}\int_V \psi(\underline{x}+\underline{\zeta},t)\gamma_f(\underline{x}+\underline{\zeta})dV_\zeta, \quad \langle\psi\rangle^f(\underline{x},t) = \frac{1}{\phi(\underline{x})}\langle\psi\rangle(\underline{x},t), \tag{2}$$

where the porosity $\phi(x)$, is defined as:

$$\phi(\underline{x}) = \frac{1}{V}\int_V \gamma_f(\underline{x}+\underline{\zeta})dV_\zeta = \frac{V_f(\underline{x})}{V}. \tag{3}$$

Volume-averaging the Navier–Stokes (N–S) equations involves integration of the space and time derivatives of quantities such as the velocity field. The volume-averaging theorem (available elsewhere e.g. [24,25]) is a useful tool in developing the governing equations for porous media flows, and it will be employed here together with the space-decomposition concept. As originally defined in Hassanizadeh and Gray [22], a quantity $\psi(\underline{r}, t)$ associated with the fluid may be decomposed in an intrinsic phase cell-average value $<\psi>^f(\underline{x}, t)$ plus a local fluctuation in space ${}^i\psi(\underline{x}, t)$:

$$\psi(\underline{r},t) = \langle\psi\rangle^f(\underline{x},t) + {}^i\psi(\underline{r},\underline{x},t). \tag{4}$$

Under the length scale constraint given by an appropriate selection of the REV, it is shown that the intrinsic phase cell-average quantity is approximately constant inside the REV [21,25]:

$$\langle{}^i\psi\rangle \cong 0, \quad \left\langle\langle\psi\rangle^f\right\rangle^f \cong \langle\psi\rangle^f. \tag{5}$$

And therefore, decomposition in Eq. (5) represents a separation of length scales where space-averaged quantities are approximately constant inside the REV.

3. Literature on turbulence models for porous media

Primary motivation behind the use of porous media models is that it is too expensive, impractical or even impossible to solve the problem in the real geometry. Therefore, equations have to be averaged in space to avoid the need to model the real geometry. Hence, distinction needs to be made between microscopic and macroscopic turbulence. The first one is turbulence detected by point-wise probes within pores, and the second one is found by averaging local turbulence in a REV. Models for the macroscopic turbulence are therefore developed by time- and space-averaging the (local) N–S equations. [It is important to mention that though time-averaging is employed in this study to Reynolds average the N–S equations [11–13], the authors recognize that time-averaging may not be appropriate when the flow is not statistically stationary (e.g. vortex shedding). Time-averaging can however be replaced by ensemble averaging without impacting the rest of the analysis.]

Unlike turbulence in clear flows, turbulence in porous media, although of practical and theoretical importance, has not been studied extensively. Not surprisingly, turbulence modeling in porous media has largely followed the modeling steps for clear flows. Consequently, the most common model for macroscopic turbulence in literature is the k–ε one, adapted to porous media [10–13]. Note that Nield [17] has discussed turbulence models for porous media in some detail. The study carried out by Antohe and Lage [11] starts from the space-averaged N–S equations for porous media (i.e., including drag forces in the momentum equation). By time-averaging these equations they developed a k–ε model for turbulence in porous media. Turbulence quantities in this study are defined by *time-averaging of the space-averaged quantities*. Their porous media model under fully developed flow conditions in one-dimension leads to the trivial solution, or $k = 0$ and $\varepsilon = 0$. Their explanation regarding the trivial solution was that, "constant level of macroscopic turbulent kinetic energy different than zero is unlikely to persist in fully developed unidirectional flow through porous media" [11].

Nakayama and Kuwahara (NK) [12] developed an alternative k–ε macroscopic model for turbulence in porous media. They noted that small eddies must be modeled first. Therefore, the modeling process in the NK model starts with the Reynolds averaged equations, which are averaged over a REV. Nakayama and Kuwahara defined the turbulence quantities by *cell-averaging the fluctuating-in-time quantities*. As in the clear flow case, their final set of equations for k and ε contains modeling constants. To determine these constants they performed 2D numerical experiments by solving the microscopic set of equations (not spatially averaged) for periodic arrays of squares. The necessary constants for the k and ε equations are then determined by averaging the results in the elementary volume. Under fully developed flow conditions in one-dimension, contrary to Antohe and Lage' model, this porous media model leads to non-trivial values for k and ε.

Pedras and de Lemos [13] addressed the issue of the order of averaging and showed that the macroscopic N–S equations for rigid porous media are independent of the order of the application of the space- and time-averaging operators (this is also addressed in [26]). Independence of the order of averaging essentially means that the time-averaging operator for rigid porous media commutes with the space-averaging operator, or:

$$\overline{\langle\psi\rangle^f}(x,t) = \frac{1}{\Delta t}\int_{\Delta t}\left[\frac{1}{V_f}\int_{V_f}\psi dV\right]d\tau = \frac{1}{V_f}\int_{V_f}\left[\frac{1}{\Delta t}\int_{\Delta t}\psi d\tau\right]dV = \langle\overline{\psi}\rangle^f(x,t). \tag{6}$$

Note that the distribution function, for the sake of simplicity in the notation, has been omitted in Eq. (6). The concept of double decomposition was also applied by Pedras and de Lemos, where any time fluctuating fluid quantity can be decomposed according to space decomposition as in Eq. (4) (<>,^{i}) or time decomposition ($\overline{\cdot}$,$'$):

$$\psi(\underline{r},t) = \langle\psi\rangle^f(\underline{x},t) + {}^i\psi(\underline{r},\underline{x},t) = \overline{\psi}(\underline{r},t) + \psi'(\underline{r},t). \tag{7}$$

Each of these components can be further decomposed. For instance, a space-averaged component may be decomposed in a space–time averaged quantity plus a time fluctuation of a space-averaged quantity.

Although not within the classical porous media context, modeling of turbulent flows through vegetation and canopies has yielded a useful description of turbulent flow in porous media [27,28]. To study flows through plant canopies, Raupach and Shaw [14] focused on the averaging procedure of local equations in two-dimensions. They identified that in addition to the space-averaged turbulent kinetic energy (TKE), conservation of kinetic energy requires the modeling of an extra term they called "*dispersive kinetic energy*". Other researchers have followed the work of Raupach and Shaw leading to transport equations for different quantities or different averages of the TKE [4,26,29]. Alternately, since the space–time-averaging of the total kinetic energy of the flow may be written as (the factor 1/2 is omitted for clarity):

$$\langle\overline{u_iu_i}\rangle^f = \overline{\langle u_i\rangle^f\langle u_i\rangle^f} + \overline{\langle{}^iu_i{}^iu_i\rangle^f} = \overline{\langle u_i\rangle^f\langle u_i\rangle^f} + \overline{\langle u_i\rangle^{f'}\langle u_i\rangle^{f'}} + \overline{\langle{}^iu_i{}^iu_i\rangle^f}, \tag{8}$$

or

$$\langle\overline{u_iu_i}\rangle^f = \langle\overline{u_i}\overline{u_i}\rangle^f + \langle\overline{u_i'u_i'}\rangle^f = \overline{\langle u_i\rangle^f\langle u_i\rangle^f} + \langle\overline{u_i'u_i'}\rangle^f + \langle\overline{{}^iu_i{}^iu_i}\rangle^f, \tag{9}$$

some models are based on the transport equations for the second term on the r.h.s. of Eq. (8) [11,26], while others are based on the second term on the r.h.s. of Eq. (9) [12,13,29]. The third term on the r.h.s. of Eq. (9) is identified as the *dispersive kinetic energy* by Raupach and Shaw [14]. In the context of porous media, this term may be called the trace of the hydrodynamic dispersion tensor of the mean flow or the trace of the momentum dispersion of the mean flow (where the word "mean" is employed to describe time averaged quantities). According to Finnigan [27], this term may be of the same order as the Reynolds stresses in the lower canopy. In the context of flows in porous media, this term intuitively seems to be important for low porosities, when the dispersion of the flow is high.

Although scarce, experimental data have been measured. Velocity profiles and turbulence statistics in canopies and vegetation are reported, for instance, in [27] and in [4], and in general porous media, in [30]. Moreover, numerical solutions of the macroscopic k–ε models have been compared with these data with acceptable agreement [4,26,30]. Microscopic numerical simulations of fully periodic REV have been mainly carried out to calculate model parameters by calculating equilibrium values [12,13,31]. However, accurate microscopic numerical simulations involving the space and time evolution of quantities of interest are difficult. This is not only because of the complexity of the geometry but also because of the difficulty to accurately model turbulent flows near the walls.

In general, turbulence models for porous media flow are developed to capture the phenomena of macroscopic turbulence by performing a space–time-averaging of the N–S equations. This procedure leads to body forces in the momentum and k–ε equations, in addition to space–time correlations, that are often neglected. For example, if decomposed using Eqs. (8) and (9), the last term in both decompositions is usually not modeled. This issue is addressed in more detail in the following section.

4. Limitations on the modeling of turbulence in porous media

Development of turbulence models for the TKE and dissipation rate in porous media that is based on averaging the local equations in space benefits by working with averages of a well known set of equations and approximations. However, the space-averaging of non-linear time averaged terms always produces additional terms involving spatial correlations. In particular, it is shown in Eqs. (8) and (9) that the space-average of the convective term in the momentum equation produces correlation terms that need to be modeled or neglected. That is, because the problem is described using the space–time average of the velocity (first term on the r.h.s. of Eqs. (8) and (9), if the macroscopic TKE (MTKE) is defined as the second term on the r.h.s. of Eqs. (8) and (9), the last term on the r.h.s. in both cases needs to be neglected in the momentum equation or included in the modeling of drag forces. Unfortunately, by choosing to work with Eqs. (8) and (9), not only the extra term in the momentum equation but additional correlations that appear in the construction of the space–time-averaging of the turbulence quantities (k and ε) also need to be modeled or neglected. Consider for instance the space-averaging of the convective term in the equation for the microscopic TKE (note that k is already a quantity averaged in time):

$$\langle \overline{u_j} k \rangle^f = \langle \overline{u_j} \rangle^f \langle k \rangle^f + \langle {}^i\overline{u_j}{}^i k \rangle^f. \tag{10}$$

The last term in Eq. (10) represents the correlation of the spatial dispersion of the time averaged velocity and the TKE. Terms of this kind appear in the formulation of the problem irrespective of the quantity chosen to model the MTKE. Appearance of such extra correlations in the averaging process is discussed in detail by Ayotte et al. [29]. The fact that these new correlations result mainly due to the way the problem is defined makes them difficult to model, and hence undesirable. The extra correlation appearing in Eq. (10) does not necessarily need to approach to zero in the limit of clear flows, when the interfacial area between both phases inside the REV is reduced to zero. Note for instance that in Nakayama and Kuwahara's model [12] this term is modeled as a diffusion term, employing the thermal dispersion tensor and the Lewis number for the mechanical dispersion. In Pedras and de Lemos's model [13] however, this term is modeled, together with the space dispersion of the production term, as a source term. These new spatial correlations that appear in the formulation are further interpreted from the energetic point of view in the following paragraphs (see for instance [14]).

Consider the momentum equation for turbulent flows in porous media. In comparison with the microscopic N–S equations, additional terms appear in the case of the porous media. These additional terms are characterized by: (1) two drag forces (form drag and viscous drag) that result from the interaction between the solid and fluid phases; (2) the flux of the hydrodynamic dispersion of the mean flow that results from the averaging of the convective term; and (3) the flux of the space average of the Reynolds stresses. The momentum equation is:

$$\frac{D}{Dt}\langle \overline{u_i} \rangle^f = -\frac{1}{\rho}\frac{\partial \langle \overline{p} \rangle^f}{\partial x_i} + \nu \frac{\partial^2 \langle \overline{u_i} \rangle^f}{\partial x_j \partial x_j} - \frac{\partial}{\partial x_j}\langle \overline{u_i' u_j'} \rangle^f - \frac{\partial}{\partial x_j}\langle {}^i\overline{u_i}{}^i \overline{u_i} \rangle^f - \frac{1}{\rho V_f}\int_{A_i} n_i \overline{p}\, dS + \frac{\nu}{V_f}\int_{A_i} n_j \frac{\partial \overline{u_i}}{\partial x_j} dS, \tag{11}$$

where the last three terms in Eq. (11) are in general modeled employing the Darcy–Forchheimer approximation (see for instance [6,12]) based on the study carried out by Vafai and Tien [32].

If Eq. (11) is multiplied by the space–time averaged velocity, the equation for the kinetic energy based on the space–time averaged velocity is obtained. In addition to the material derivative of this quantity and a transport term, this equation has several negative sources (or "sinks") of energy as can be seen by inspection of Eq. (11). These negative sources are (for simplicity the constant porosity case is considered):

$$Sinks = \langle \overline{u_i' u_j'} \rangle^f \frac{\partial \langle \overline{u_i} \rangle^f}{\partial x_j} + \langle {}^i\overline{u_i}{}^i \overline{u_i} \rangle^f \frac{\partial \langle \overline{u_i} \rangle^f}{\partial x_j} + \langle \overline{u_i} \rangle^f \left[-\frac{1}{\rho V_f}\int_{A_i} n_i \overline{p}\, dS + \frac{\nu}{V_f}\int_{A_i} n_j \frac{\partial \overline{u_i}}{\partial x_j} dS \right] - \nu \frac{\partial \langle \overline{u_i} \rangle^f}{\partial x_j}\frac{\partial \langle \overline{u_i} \rangle^f}{\partial x_j}. \tag{12}$$

The first three terms on the r.h.s. of Eq. (12) are actually responsible for the transfer of kinetic energy to the filtered motions but they cannot change the kinetic energy of the flow. In other words, these three terms must appear as source terms in the equations for the filtered motions. This can be understood considering the energy equation for the microscopic flow [33]:

$$\frac{DE}{Dt} + \nabla \cdot T = -E, \tag{13}$$

where the flux of energy and the viscous dissipation are defined as:

$$T_j = \frac{\delta_{ij} u_i p}{\rho} - \nu u_i \frac{\partial u_i}{\partial x_j}, \quad E = \nu \frac{\partial u_i}{\partial x_j}\frac{\partial u_i}{\partial x_j}. \tag{14}$$

Hence, if the energy equation is integrated over a REV of a porous medium, as T is identically zero over the solid–fluid interface, neither drag forces nor velocity correlations can create/destroy kinetic energy. (This point is also discussed by de Lemos and Pedras in Ref. [16].) Nevertheless, if filtered motions are considered, those can produce turbulent kinetic energy or dispersive kinetic energy. In other words, residual motion can be produced by drag forces or velocity correlations (see for instance [14]). The last term in Eq. (10) is a clear example of such a term. It can be interpreted as a

transfer of turbulent kinetic energy into dispersive kinetic energy and naturally, it must also appear with opposite sign in the transport equation for the dispersive kinetic energy to assure the conservation of the kinetic energy.

The first term on the r.h.s. of Eq. (12) or production of TKE (with a negative sign) is the only term that appears as a positive source term in the transport equation for the space-averaged turbulent kinetic energy of the flow [12,13]. Thus, the remaining negative sources in Eq. (12) (with the exception of the dissipation) must cause the dispersion of the flow in the porous medium and it is of interest to include them in the modeling process of the average of the TKE. Therefore, one of the objectives of a new model for turbulent flows in porous media may then be to capture the entire energy filtered in the space–time-averaging process. This motivates the development presented in the following section

5. Model

In this study, a new formulation to better represent the phenomenon of turbulence in porous media is developed. Following an approach similar to that followed to obtain existing turbulence models, a new two-equation k–ε type model is developed. However, considering the limitations of the existing turbulence models for flow in porous media (see Section 4), the model being developed here is expected to have the following characteristics:

- It should model the entire kinetic energy filtered in the averaging process.
- It should not neglect any terms in the development.
- All terms in the final form of the k and ε equations, in addition to the terms similar to those that appear in the corresponding equations for clear flow, should result from the interaction between the solid and fluid phases. Hence, such terms should naturally approach zero in the limit of clear flow.

Such a formulation is expected to be simpler and also contribute to a better understanding of the problem. In this regard, the fact that the problem of turbulence in porous media may be described by quantities averaged in space and in time, together with the fact that distinction between space fluctuation and time fluctuation yield additional correlations, suggest the possibility of developing a model in which distinction is not made between space-and-time averaged quantities based on the order of averaging/integration. Hence, in this model, the averaged and fluctuating quantities are defined as before, but the order of the steps followed in the averaging procedure of a fluid phase variable is not critical. Any quantity associated with the fluid is then decomposed as a space–time averaged quantity plus a fluctuation:

$$\psi(\underline{r},t) = \overline{\overline{\psi}}(\underline{x},t) + \psi''(\underline{r},\underline{x},t), \tag{15}$$

where

$$\overline{\overline{\psi}}(\underline{x},t) = \frac{1}{\Delta t}\int_{\Delta t}\left[\frac{1}{V_f}\int_{V_f}\psi dV\right]d\tau = \frac{1}{V_f}\int_{V_f}\left[\frac{1}{\Delta t}\int_{\Delta t}\psi d\tau\right]dV, \tag{16}$$

and

$$\overline{\overline{\overline{\psi}}} \cong \overline{\overline{\psi}}, \quad \overline{\overline{\psi''}} \cong 0. \tag{17}$$

Note that in Eqs. (15)–(17), double-bars and double-primes have been used to describe space–time-averaging and deviations from this double-averaged quantity, respectively. It is important to note again that Eq. (15) represents a decomposition of length scales, with $\overline{\overline{\psi}}$ being approximately constant inside the REV (this is also expressed in Eq. (17)).

5.1. Natural definition of the macroscopic turbulence kinetic energy (k)

Following the definition of the fluctuation with respect to a space–time averaged quantity, the average of the convective term in the momentum equation leads to the following additional term:

$$\overline{\langle u_i'' u_i''\rangle^f} = \overline{\overline{u_i'' u_i''}}, \tag{18}$$

therefore suggesting an appropriate definition of the macroscopic turbulence kinetic energy (k) to be:

$$k = \frac{\overline{\overline{u_j'' u_j''}}}{2}. \tag{19}$$

This quantity will be called the macroscopic TKE (MTKE). More accurately, this quantity represents *all* the energy filtered in the averaging process.

Different turbulence models for porous media have used different definitions of the MTKE. The relationship between them has been stated for instance in Pedras and de Lemos [15]. The relation between the quantity introduced here and previous definitions (k_{NK} for NK's model, and k_M, the definition adopted by Antohe and Lage [11]) is:

$$2k = \langle \overline{{}^i u_i {}^i u_i}\rangle^f + 2k_{NK} = \langle \overline{{}^i u_i {}^i u_i}\rangle^f + \langle \overline{{}^i u_j' {}^i u_j'}\rangle^f + 2k_M. \tag{20}$$

This equation shows that the MTKE, as it is defined here, includes the trace of the hydrodynamic dispersion of the mean flow (also called *dispersive kinetic energy* [14]). Although this term is different from zero even when turbulence vanishes (i.e., in laminar flow), it seems appropriate to include it in the definition of the MTKE. This is not only because the final set of equations based on this definition of the MTKE shows several desirable properties but also because the resulting model seeks to capture all of the kinetic energy filtered in the averaging process. Note that the MTKE defined here is different from zero in the laminar case. Due to its similarity with corresponding definitions in porous media models it is still called MTKE as it includes the turbulence behavior at high *Re* numbers. Moreover, the analysis that follows is for turbulent flow where the Reynolds averaging is needed.

The definition of the MTKE introduced here, which is different from those used by other researches, makes it impossible to use the space average of the corresponding microscopic equations to derive the turbulence model. It is however still possible to develop the transport-diffusion equations for the redefined quantities by applying the procedure commonly followed in the development of turbulence models for clear flows. Before proceeding with the development of the turbulence model, some rules useful in performing the averaging of the equations are summarized here.

5.2. Averaging rules

Applying the theorem of volume-averaging [24,25] to calculate the average of the gradient of a constant scalar c, shows that for constant porosity, the area integral of the normal vector is equal to zero (see for instance [21]):

$$\overline{\overline{\nabla c}}^v = 0 = \nabla \overline{\overline{c}}^v + \frac{1}{V}\int_{A_i} c n_j dA = c\nabla\phi + \frac{c}{V}\int_{A_i} n_j dA = \frac{c}{V}\int_{A_i} n_j dS = 0. \tag{21}$$

Note that the superscript "v" is introduced to denote cell-averaging.

In common turbulence models, velocity fluctuations at the wall are identically zero because of the no-slip condition. With the definition of velocity fluctuations introduced here (Eq. (15)), this is not true because the actual velocity at the wall is written in terms of

the space–time averaged velocity over the REV. However, although the fluctuations are not zero at the wall, they have a spatially constant value inside the REV. This can be understood from the definition of space–time fluctuations considering the value of the velocity over all walls inside a particular REV:

$$U_i''(\underline{x}, \underline{r}_{wall}, t) = -\overline{\overline{u}_i}(\underline{x}, t). \tag{22}$$

With the use of these rules, (Eqs. (21) and (22)), it is possible to show that the following results hold for *constant porosity*:

$$\overline{\overline{\frac{\partial}{\partial x_j} u_i''(\underline{x}, \underline{x}+\underline{\zeta}, t)^v} = \frac{\partial}{\partial x_j}\phi \underbrace{\overline{\overline{u_i''(\underline{x}, \underline{x}+\underline{\zeta}, t)}}}_{=0} + \frac{1}{V\Delta t}\int_{\Delta t}\int_{A_i} n_j u_i''(\underline{x}, \underline{r}_{wall}, \tau) dS = 0, \tag{23}$$

and

$$\overline{\overline{\frac{\partial}{\partial x_j}\overline{\overline{u}_i}(\underline{x}, t)^v} = \frac{\partial}{\partial x_j}\phi\overline{\overline{u}_i}(\underline{x}, t) + \frac{\overline{\overline{u}_i}(\underline{x}, \tau)}{V}\underbrace{\int_{A_i} n_j dS}_{=0} = \frac{\partial}{\partial x_j}\phi\overline{\overline{u}_i}(\underline{x}, t). \tag{24}$$

5.3. Macroscopic equations

In the following paragraphs, the averaging of the N–S equations is carried out by using the theorem of volume-averaging, and rules stated in Eqs. (21)–(24). The construction of k and ε equations is carried out following the corresponding steps for clear flows (available elsewhere e.g. [34]). The derivation of the corresponding equations for the Reynolds stresses and macroscopic turbulent quantities is not complex. However, it is long and tedious. For this reason, all intermediate steps are not presented here; only some of them are highlighted (for a complete description of the derivation of macroscopic equations, see [19]). For the sake of simplicity in the notation, from now on a *single bar on top* means *time–space-averaging* (defined in Eqs. (15) and (16)).

In order to keep the derivation simple and to facilitate easy interpretation of the final results, the medium is considered isotropic with constant porosity, with incompressible and isothermal flow. The assumption of constant porosity simplifies the development of the equations as can be guessed by inspection of Eqs. (23) and (24), but is not restrictive at all. Under these assumptions, the resulting space–time averaged equations for an incompressible fluid flowing through an isotropic, constant porosity medium are:

Mass conservation:

$$\frac{\partial \overline{u_i}}{\partial x_i} = 0. \tag{25}$$

Momentum conservation:

$$\frac{\partial \overline{u_i}}{\partial t} + \frac{\partial}{\partial x_j}(\overline{u_i}\overline{u_j} + \overline{u_i''u_j''}) = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_i} + \nu\frac{\partial^2 \overline{u_i}}{\partial x_j \partial x_j} - \frac{1}{\rho}\frac{1}{\Delta t V_f}\int_{\Delta t} d\tau \int_{A_i} n_i p dS + \frac{\nu}{\Delta t V_f}\int_{\Delta t} d\tau \int_{A_i} n_j \frac{\partial u_i}{\partial x_j} dS. \tag{26}$$

The equation for the fluctuations is found by subtracting the averaged momentum equation from the momentum equation,

$$\frac{\partial u_i''}{\partial t} + \frac{\partial}{\partial x_j}(\overline{u_j}u_i'' + \overline{u_i}u_j'' + u_i''u_j'' - \overline{u_i''u_j''}) = -\frac{1}{\rho}\frac{\partial p''}{\partial x_i} + \nu\frac{\partial^2 u_i''}{\partial x_j \partial x_j} + R_i, \tag{27}$$

with

$$R_i = \frac{1}{\rho}\frac{1}{\Delta t V_f}\int_{\Delta t} d\tau \int_{A_i} n_i p dS - \frac{\nu}{\Delta t V_f}\int_{\Delta t} d\tau \int_{A_i} n_j \frac{\partial u_i}{\partial x_j} dS. \tag{28}$$

Note that R_i, that also appears in the momentum equation and results from the interaction between the porous media walls and the fluid, is a quantity already averaged in time and space, i.e., it is constant over the microscopic scale and has no time fluctuating component.

5.3.1. Macroscopic turbulent kinetic energy (k) equation

The equation for the *Reynolds stresses* is derived by calculating the quantity:

$$\overline{u_i'' x\text{Eq. (39)}_k + u_k'' x\text{Eq. (39)}_i}. \tag{29}$$

In the evaluation of Eq. (29), for instance, the viscous term can be calculated as follows

$$\begin{aligned}
\nu\overline{\left[u_k''\frac{\partial^2 u_i''}{\partial x_j \partial x_j} + u_i''\frac{\partial^2 u_k''}{\partial x_j \partial x_j}\right]} &= \nu\left[\overline{\frac{\partial}{\partial x_j}\left(u_k''\frac{\partial u_i''}{\partial x_j}\right)} - 2\overline{\frac{\partial u_i''}{\partial x_j}\frac{\partial u_k''}{\partial x_j}} + \overline{\frac{\partial}{\partial x_j}\left(u_i''\frac{\partial u_k''}{\partial x_j}\right)}\right] \\
&= \nu\left[\frac{\partial}{\partial x_j}\overline{\frac{\partial}{\partial x_j}(u_i''u_k'')} + \frac{1}{\Delta t V_f}\int_{\Delta t} d\tau \int_{A_i} n_j \frac{\partial}{\partial x_j}(u_i''u_k'') dS\right. \\
&\left. -2\overline{\frac{\partial u_i''}{\partial x_j}\frac{\partial u_k''}{\partial x_j}}\right] = \nu\left[\frac{\partial^2}{\partial x_j \partial x_j}(\overline{u_i''u_k''}) - 2\overline{\frac{\partial u_i''}{\partial x_j}\frac{\partial u_k''}{\partial x_j}}\right] \\
&+ \frac{\nu}{\Delta t V_f}\int_{\Delta t} d\tau \int_{A_i} n_j \frac{\partial}{\partial x_j}(u_i''u_k'') dS.
\end{aligned} \tag{30}$$

One of the integrals terms in Eq. (30) is zero for the constant porosity case. The term involving the drag forces is given by,

$$\overline{u_k''R_i} + \overline{u_i''R_k} = \overline{u_k''}R_i + \overline{u_i''}R_k = 0. \tag{31}$$

The final form of the equation for the MTKE is (trace of Eq. (29)):

$$\begin{aligned}
\frac{\partial k}{\partial t} + \overline{u_j}\frac{\partial k}{\partial x_j} &= -\overline{u_j''u_i''}\frac{\partial \overline{u_i}}{\partial x_j} - \frac{\partial}{\partial x_j}\left[\frac{1}{\rho}(\overline{p''u_j''}) + \frac{1}{2}\overline{u_i''u_i''u_j''}\right] + \nu\frac{\partial^2 k}{\partial x_j^2} - \varepsilon \\
&- \overline{u_i}\left[-\frac{\overline{u_i}}{\rho\Delta t V_f}\int_{\Delta t} d\tau \int_{A_i} n_i p'' dS + \frac{\nu\overline{u_i}}{\Delta t V_f}\int_{\Delta t} d\tau \int_{A_i} n_j \frac{\partial u_i''}{\partial x_j} dS\right],
\end{aligned} \tag{32}$$

where k, the MTKE, has already been defined in Eq. (19). Eq. (32) is the same as the corresponding equation in clear flow with two additional body force terms. One interesting feature to note is that the body forces appearing in the k-equation are the same as those found in the momentum equation multiplied by the averaged velocity. Moreover, this term was a sink term in the equation for the kinetic energy of the space–time averaged flow (see Eq. (12)) and thus, it must appear as a source term in the equation for the filtered kinetic energy.

5.3.2. Macroscopic dissipation rate (ε) equation

Note that the macroscopic dissipation rate (MDR, ε) has already been introduced in Eq. (32). It appears from the averaging of the viscous term (Eq. (30), with indices $k = i$). In the context of the approach followed here its natural definition is:

$$\varepsilon = \nu\overline{\frac{\partial u_i''}{\partial x_j}\frac{\partial u_i''}{\partial x_j}}. \tag{33}$$

To derive the governing equation for the macroscopic dissipation rate (ε), the equation for the space–time fluctuation (Eq. (27)) is operated by the following operator,

$$\overline{2\nu\frac{\partial u_i''}{\partial x_k}\frac{\partial}{\partial x_k}}. \tag{34}$$

The pressure term in Eq. (27) in the ensuing derivation is averaged as:

$$\overline{\frac{2\nu}{\rho}\frac{\partial u_i''}{\partial x_k}\frac{\partial}{\partial x_k}\frac{\partial p''}{\partial x_i}} = \frac{2\nu}{\rho}\left[\frac{\partial}{\partial x_i}\overline{\left[\frac{\partial u_i''}{\partial x_k}\frac{\partial p''}{\partial x_k}\right]} + \frac{1}{\Delta t V_f}\int_{\Delta t} d\tau \int_{A_i} n_i \frac{\partial u_i''}{\partial x_k}\frac{\partial p''}{\partial x_k} dS\right]. \tag{35}$$

where it is recognized that the divergence of the velocity fluctuations is zero for the constant porosity case. The final form of the equation for the dissipation rate is:

$$\begin{aligned}\frac{\partial \varepsilon}{\partial t} + \overline{u_j}\frac{\partial \varepsilon}{\partial x_j} = {} & \nu\frac{\partial^2 \varepsilon}{\partial x_j \partial x_j} - \frac{\partial}{\partial x_j}\left[\overline{u_j'' \varepsilon_m} + \frac{2\nu}{\rho}\overline{\frac{\partial u_j''}{\partial x_k}\frac{\partial p''}{\partial x_k}}\right] \\ & - 2\nu^2 \overline{\frac{\partial^2 u_i''}{\partial x_j x_k}\frac{\partial^2 u_i''}{\partial x_j x_k}} - 2\nu\left[\overline{\frac{\partial u_i''}{\partial x_k}\frac{\partial u_i''}{\partial x_j}\frac{\partial \overline{u_j}}{\partial x_k}} + \overline{\frac{\partial u_i''}{\partial x_k}\frac{\partial u_j''}{\partial x_k}\frac{\partial \overline{u_i}}{\partial x_j}} + \overline{u_j''\frac{\partial u_i''}{\partial x_k}}\frac{\partial^2 \overline{u_i}}{\partial x_j \partial x_k}\right. \\ & \left. + \overline{\frac{\partial u_i''}{\partial x_k}\frac{\partial u_i''}{\partial x_j}\frac{\partial u_j''}{\partial x_k}}\right] - \frac{2\nu}{\rho \Delta t V_f}\int_{\Delta t} d\tau \int_{A_i} n_i \frac{\partial u_i''}{\partial x_k}\frac{\partial p''}{\partial x_k} dS + \frac{\nu}{\Delta t V_f} \\ & \times \int_{\Delta t} d\tau \int_{A_i} n_j \frac{\partial \varepsilon_m}{\partial x_j} dS + \nu\frac{\partial}{\partial x_j}\left[\frac{1}{\Delta t V_f}\int_{\Delta t} d\tau \int_{A_i} n_j \varepsilon_m dS\right].\end{aligned} \tag{36}$$

This is again the same as the corresponding equation for clear flows, with three additional boundary terms. Note that the second term on the r.h.s. of Eq. (36) is known as the diffusion term, the third term as the destruction term and the fourth term as the production term. The symbol ε_m is used to describe the microscopic dissipation rate ($\varepsilon = \overline{\varepsilon_m}$).

6. Model closure

In the set of equations developed here for turbulent flows in porous media several terms need to be modeled. As in clear flows, closures for the second and third order one-point correlations appearing in the turbulence transport equations are needed. These correlations, that in the present case involve space fluctuations, and are averaged over space, are modeled assuming a natural extension from the k–ε model for the clear flow case to the macroscopic description of flow in porous media. The same approach has also been followed in literature (e.g. [12,13]) and is largely dictated by the lack of experimental and/or numerical data for porous media flows. It is however also recognized that this assumption needs to be validated. In addition, terms specific to the porous media, such as the drag force in the momentum equation and the integral terms that appear in the turbulence transport equations need to be modeled. The drag term in the momentum equation is generally modeled using the Darcy–Forchheimer approximation [1,32]. With this approximation for the drag force, integral terms in the k-equation can be easily modeled. Integral terms in the ε-equation require further investigation to develop an adequate model. Closure equations for a one-equation and a two-equation model are described below.

6.1. One-equation model

Based on the discussion above, a one-equation model for turbulent flow in porous media can be readily developed. The usual procedure and approximations [35] lead to the following model for mass, momentum and MTKE conservation:

$$\frac{\partial \overline{u_i}}{\partial x_i} = 0, \tag{37}$$

$$\frac{D\overline{u_i}}{Dt} = -\frac{1}{\rho}\frac{\partial(\overline{p} + 2/3k)}{\partial x_i} + \frac{\partial}{\partial x_j}\left[(\nu + \nu_T)\frac{\partial \overline{u_i}}{\partial x_j}\right] + R_i, \tag{38}$$

$$\frac{Dk}{Dt} = -\overline{u_j'' u_i''}\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_T}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] - \varepsilon - \overline{u_i} R_i, \tag{39}$$

with:

$$R_i = -\frac{\phi\nu}{K}\overline{u_i} - \frac{\phi^2 F}{\sqrt{K}}\sqrt{\overline{u_j u_j}}\,\overline{u_i}, \quad \overline{u_i'' u_j''} = -\nu_T\left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i}\right) + \frac{2k}{3}\delta_{ij}, \tag{40}$$

$$\varepsilon = C_D\frac{k^{3/2}}{l_m} \quad \text{and} \quad \nu_T = c\,k^{1/2} l_m, \tag{41}$$

where the first equation in (40) represents the Darcy–Forchheimer approximation that depends upon the permeability K and Forchheimer constant F [1], and l_m is the mixing length scale (C_D and c are model constants, see for instance Pope [33]).

6.2. Two-equation model

Additional closure issues must be addressed to close the two-equation model. Specifically, models for the boundary terms in the macroscopic dissipation rate Eq. (36) must be developed. These boundary terms act as a source term enhancing the dissipation rate

$$\begin{aligned}& -\frac{2\nu}{\rho \Delta t V_f}\int_{\Delta t} d\tau \int_{A_i} n_i \frac{\partial u_i''}{\partial x_k}\frac{\partial p''}{\partial x_k} dS + \frac{\nu}{\Delta t V_f}\int_{\Delta t} d\tau \int_{A_i} n_j \frac{\partial \varepsilon_m}{\partial x_j} dS \\ & + \nu\frac{\partial}{\partial x_j}\left[\frac{1}{\Delta t V_f}\int_{\Delta t} d\tau \int_{A_i} n_j \varepsilon_m dS\right] = \textit{Source term}.\end{aligned} \tag{42}$$

The dispersion produced by the solid phase acting on the fluid phase creates MTKE (walls producing turbulence). MDR is also enhanced to dissipate this energy. These boundary integrals can be estimated from experimental results or numerical simulations that accurately predict the near wall behavior of the relevant quantities. However, because such data for porous media are still not readily available, a simple model based on physical insight is proposed here.

To start, it is useful to consider the model characteristics in one-dimensional flow in isotropic, constant porosity, porous media. Contrary to the conditions found in clear flows, where isotropic turbulence decays away, the interaction between the solid and the fluid phases in porous media maintains a constant level of the turbulence quantities and therefore a constant level of the macroscopic turbulence quantities. This expectation is fulfilled by the momentum and the macroscopic k-equation. For 1D, fully developed flow in isotropic media, Eqs. (38) and (39) reduce to:

$$\varepsilon = -\overline{u_i} R_i = \frac{\overline{u_i}}{\rho}\frac{\partial \overline{p}}{\partial x_i}. \tag{43}$$

Eq. (43) is simply a statement of the conservation of kinetic energy, meaning that, for constant kinetic energy, the work done by the pressure must be dissipated by the viscosity.

The Darcy–Forchheimer approximation for the case of turbulent flows becomes primarily an inertial force and is proportional to the square of the velocity (see Eq. (40)). This implies that the MDR scales with the cube of the intrinsic fluid velocity U, while the square root of the permeability provides the length scale:

$$\varepsilon \approx \frac{U^3}{\sqrt{K}}. \tag{44}$$

In addition, it is known that the MTKE scales with the square of the velocity [12,13]. Therefore, inspection of the dissipation equation for the case under consideration leads to:

$$\frac{\varepsilon^2}{k} \approx \frac{U^6}{K}\frac{1}{U^2} = \frac{U^4}{K} \approx \textit{boundary terms}. \tag{45}$$

Boundary terms must approach zero in the limit of no interfacial area and therefore they may be modeled using the inverse of the permeability. Although the fluid viscosity appears in the boundary terms under consideration, it is known that the dissipation in the limit of zero viscosity remains at a finite value, thus

implying that the boundary terms may remain also finite in this limit. For this reason, it seems realistic to model the interfacial integrals in the dissipation equation in terms of the following set of variables: k, ε, U and K. Because it has been already assumed that k scales with the square of the velocity, the attention is directed to ε, U and K variables. The following dimensional analysis for the boundary term,

$$\frac{U^4}{K} \approx F(\varepsilon, U, K), \tag{46}$$

coupled with the dual requirement that the MDR appears in the model and that it, together with the velocity, appear in the numerator, leads to:

$$\textit{boundary terms} \approx \frac{\varepsilon U}{\sqrt{K}}. \tag{47}$$

In accordance with the scale analysis given above, the model chosen for the boundary terms in isotropic media is,

$$\textit{boundary terms} = f(\phi, K)\frac{\varepsilon\sqrt{\overline{u_j}\,\overline{u_j}}}{\sqrt{K}}, \tag{48}$$

where a function of porosity and permeability, $f(\phi, K)$, has been introduced to account for the morphology of the medium. With the proposed model for the boundary terms, the two-equation model is now complete.

The additional equations that complete the set described in Eqs. (37)–(40) are:

$$\frac{D\varepsilon}{Dt} = -C_{\varepsilon 1}\frac{\varepsilon}{k}\overline{u_j'' u_i''}\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial}{\partial x_j}\left[\left(\frac{\nu_T}{\sigma_\varepsilon} + \nu\right)\frac{\partial \varepsilon}{\partial x_j}\right] - C_{\varepsilon 2}\frac{\varepsilon^2}{k} + f(\phi, K)\frac{\varepsilon\sqrt{\overline{u_j}\,\overline{u_j}}}{\sqrt{K}}, \tag{49}$$

$$\nu_T = C_\mu \frac{k^2}{\varepsilon}. \tag{50}$$

To gain some physical insight from the final set of equations, it is again useful to consider their behavior for 1D, fully developed flow in isotropic constant porosity porous media. In this case, the steady-state values of the turbulence quantities are,

$$\varepsilon = \frac{\phi\nu}{K}|\overline{u}|^2 + \frac{\phi^2 F}{\sqrt{K}}|\overline{u}|^3, \tag{51}$$

$$k = \frac{C_{\varepsilon 2}}{f(\phi, K)}\left(\frac{\phi\nu}{\sqrt{K}}|\overline{u}| + \phi^2 F|\overline{u}|^2\right). \tag{52}$$

Here, the Darcy (linear) term in the Darcy–Forchheimer approximations has been purposely retained in the equations. As the averaging process is still valid for laminar flows and the Darcy term captures the flow behavior in this regime, it is desirable to retain this term to study laminar flow. However, it is important to remember that the modeling of the boundary terms in the dissipation equation is based on inertial flow considerations more than Stokes type flows. Additional assumptions based on turbulent flow considerations, regarding the turbulent mixing length scale, will be introduced later.

Recalling the function $f(\phi, K)$, it is noted that based on the fact that MTKE must go to zero in the limit of clear flows (when $\phi \to 1$ and $K \to \infty$), Eq. (52) imposes a mathematical condition on the asymptotic behavior of the function. That is, $f(\phi, K)$ must tend to infinity as the porosity approaches 1 and the permeability goes to infinity. In addition, the representation of integral terms in Eq. (48) implies that $f(\phi, K)$ must increase as $\phi \to 1$ and $K \to \infty$ slower than the square root of permeability to ensure that the model proposed for the integral terms vanishes in the limit of clear flows. Although this function needs to be determined from microscopic calculations in representative cells of an infinite periodic porous media, it is of interest to note that one-equation and two-equation models lead to the same result for the steady-state value of k if $f(\phi, K)$ is defined as:

$$f(\phi, K, Re_{\sqrt{K}}) = C_{\varepsilon 2}C_D^{2/3}\left(\frac{\phi}{Re_{\sqrt{K}}}\frac{K}{l_m^2} + \phi^2 F\frac{K}{l_m^2}\right)^{1/3}, \tag{53}$$

where, in this case, $f(\phi, K)$ may be considered also to be a function of a local Reynolds number that results from the Darcy term in the Darcy–Forchheimer approximation. This definition of f fulfills the required physical conditions, if the mixing length scale remains finite, in the limit of clear flows.

7. Discussion and summary

A new k–ε model to describe the phenomenon of turbulence in porous media has been developed. The space–time fluctuation in this model is treated as a unique quantity and a transport-diffusion equation has been derived here for the *entire* kinetic energy filtered in the averaging process of the momentum equation. A simple closure model was assumed to complete the one- and two-equation models. For a given porous medium, the one-equation model is closed once a mixing length scale and model constants are defined (Eqs. (37)–(41)). The two-equation model needs additional information such as the function f (Eq. (49)) or equivalently, the fully developed value of the MTKE (Eq. (52)), for closure.

The assumption of constant porosity may seem restrictive. However, the steps followed here to develop the equations do not restrict the development to this limiting case. The non-uniform porosity case would lead to additional boundary terms as can be guessed by inspection of Eqs. (23) and (24). Since information regarding the non-uniform porosity model is usually lacking, modeling with the additional complexity due to a variable porosity is not warranted at this stage.

An important aspect in turbulence modeling is the treatment of the transfer of kinetic energy between the space-averaged kinetic energy and the residual energy [33]. The fact that space and time fluctuations in the model proposed here are not explicitly distinguished allows the development of a transport equation for the entire filtered energy. This also eliminates space correlations of time averaged values and time correlations of space-averaged values from the final set of equations. Moreover, the body force appearing in the k-equation results naturally from the averaging process defining macroscopic dissipation at equilibrium as the product of the pressure gradient and velocity.

In comparison with the clear flow case, the interaction between walls and the fluid in porous media produces additional terms (or area integral terms) in the final set of equations. Models proposed for these terms include as a factor, the inverse of permeability to ensure that they vanish as the surface area of the solid tends to zero. However, terms that arise from the space-averaging of nonlinear terms (see Section 4, Eq. (10)) may not vanish in the limit of clear flows and therefore they may not be modeled using the inverse of permeability as a factor. In this regard, it is of interest to consider the form of the averaged equations when porosity approaches one. Not only in the model proposed here, but also in models that average the microscopic TKE, the averaged equations do not need to recover the clear flow equations in the limit of porosity approaching 1. Clear flow equations are recovered in the limit of zero REV's volume.

The dissipation rate equation in porous media has been developed, for instance, for the space average of the local dissipation [13,29] or proposed as an empirical transport-diffusion equation that is obtained, in analogy with clear flows, by rescaling the equation for the MTKE with ε/k [26]. This leads to several new space–time correlations that need to be modeled. However, the equation

for the macroscopic dissipation rate derived here following a rather *systematic* approach leads to correlations already found in the corresponding clear flow equation. The only difference between the porous media and the corresponding clear flow equations are the integral terms in the porous media equation that vanish in the limit of clear flows. These boundary terms need to be studied in detail by means of experiments or direct numerical simulations of the N–S equations. To close the set of equations an empirical correlation based on dimensional analysis and consistent with the TKE equation is proposed.

The model presented in this study is based on a new definition for macroscopic turbulence quantities. These entities need to be analyzed and compared with existing definitions under different flow conditions and different porosities to understand their physical significance. To validate the model developed here it is solved numerically and the results are presented in a companion paper [20]. Results are compared with those available in literature.

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